EFFECT OF CONDUCTOR THICKNESS ON THE MODE CAPACITANCES OF SHIELDED STRIP TRANSMISSION LINES

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ABSTRACT

The charge distribution on shielded stripline conductors of finite thickness is approximated by a numerical integration technique. A new model has been devised to describe very thin strips. The effects of thickness may be significant.

Summary

A cross section of the strip transmission line to be considered is shown in Figure 1. The structure is filled with a uniform dielectric material. In general the strip conductors are not constrained to lie in the same plane, but may be overlapped, as shown. This is a nondispersive structure that

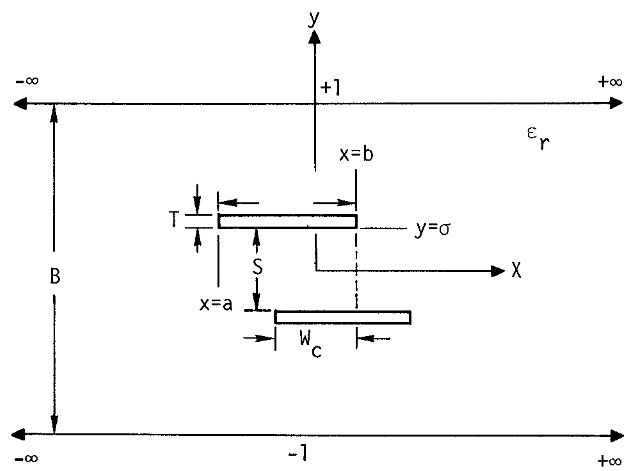

Figure 1. Cross-sectional view of coupled overlapped strip transmission lines

will support a TEM transmission line mode. The general analysis scheme described here is that of Chestnut.[1] The potential at any point in the cross-sectional plane may be expressed as:

$$\phi\,(x,y) = \oint G \frac{\partial \phi}{\partial n}\, d\ell' \tag{1}$$

where the integration is along the surfaces of the strip conductors and the Green's function

$$G(x,y;x',y') = -\frac{1}{4\pi}\,\ell n\left[\frac{\cosh\frac{\pi}{2}\,(x-x')-\cos\frac{\pi}{2}\,(y-y')}{\cosh\frac{\pi}{2}\,(x-x')+\cos\frac{\pi}{2}\,(y+y')}\right] \tag{2}$$

As the field points merge with the source points,

$$(x,y) \rightarrow (x',y') \tag{3}$$

the following singular expression is used

$$G(x,y;x',y') \rightarrow -\frac{1}{2\pi}\,\ell n\,\sqrt{(x-x')^2+(y-y')^2} \tag{4}$$

The quantity $(\epsilon\,\frac{\partial \phi}{\partial n})$ represents the surface charge density on the strips. The potential of the strips is considered to be either ± 1 volt, depending on the mode of excitation, and hence Equation (1) may be written:

$$\begin{aligned}1 = &\int_a^b \left[G(x,y;x',\sigma)\ \pm G(x,y;-x',-\sigma)\right] \frac{\partial \phi(x',\sigma)}{\partial y}\, dx' \\ &- \int_a^b \left[G(x,y;x',\sigma+T)\ \pm G(x,y;-x',-\sigma-T)\right] \frac{\partial \phi(x',\sigma+T)}{\partial y}\, dx' \\ &- \int_\sigma^{\sigma+T} \left[G(x,y;a,y')\ \pm G(x,y;-a,-y')\right] \frac{\partial \phi(a,y')}{\partial x}\, dy' \\ &- \int_\sigma^{\sigma+T} \left[G(x,y;b,y')\ \pm G(x,y;-b,-y')\right] \frac{\partial \phi(b,y')}{\partial x}\, dy\end{aligned} \tag{5}$$

where the $\pm$ sign indicates even or odd excitation, respectively. The capacitances of the coupled lines are

$$C_{e,o} = \epsilon \int \frac{\partial \phi_{e,o}}{\partial n}\, d\ell' \tag{6}$$

for the even and odd modes. The integration is around the strip periphery.

If the strip thickness is considered negligible, i.e., T=0, then Equation (1) becomes

$$1 = \int_a^b \left[G(x,\sigma;x',\sigma)\ \pm G(x,\sigma;-x'-\sigma)\right]\left[\frac{\partial \phi(x',\sigma^-)}{\partial y} - \frac{\partial \phi(x',\sigma^+)}{\partial y}\right] dx' \tag{7}$$

The singularity in any of the integrands may be handled in the manner suggested by Chestnut[1]. Each integral is of the form

$$\int_a^b K(x,t)f(t)dt \tag{8}$$

and may be decomposed by quadrature to

$$\sum_{j=1}^{N} w_j\ K(x,t_j)f(t_j) \tag{9}$$

When $x=t_j$ the following form of Equation (6) is used

$$\int_a^b \left[K(x,t) + \frac{1}{2\pi}\ \ln|x-t|\right]f(t)dt$$

$$- \frac{1}{2\pi}\int_a^b \ln|x-t|[f(t)-f(x)]dt$$

$$- \frac{1}{2\pi}\ f(x)\int_a^b \ln|x-t|dt \qquad (10)$$

By evaluating quadrature approximations to Equations (5) or (7) at points on the conductor surface, a system of linear equations is generated from which surface charge density may be solved.

The approach used to test the accuracy of the computational method was to compare resulting solutions with known rigorous solutions. Exact rigorous solutions are available through conformal mapping procedures for the centered single conductor of infinite thinness and for two coplanar conductors of infinite thinness. Excellent agreement with the exact solution was found as indicated in Tables 1 and 2. Good agreement was obtained with Shelton's[2] approximate solutions for offset strips of zero thickness, with the best agreement occurring within Shelton's region of maximum accuracy.[3] The degree of excellence of agreement in all cases increased with N, the number of Gaussian points, hence an extrapolation scheme was employed to accomplish a large effective value of N.

Table 1. Results from Analysis for Single Strip Stripline[a]

W/B	C/ε, Exact	C/ε, Extrapolated	Percent Difference
0.0001	0.61933	0.61933	---
0.0003	0.69455	0.69455	---
0.001	0.80117	0.80117	---
0.003	0.93169	0.93169	---
0.01	1.13417	1.13417	---
0.03	1.41467	1.41467	---
0.1	1.93965	1.93964	-0.0005
0.3	2.91347	2.91346	-0.0003
1.0	5.76449	5.76446	-0.0005

[a]Percent Difference = $\frac{\text{C, Extrapolated - C, Exact}}{\text{C, Exact}}$ x 100.

Table 2. Comparison of Approximate and Exact Values of Even and Odd Mode Capacitances of Coplanar Coupled Strip Conductors (W_c/B=-0.1)[a]

W/B	Exact Values		Approximate Values			
	C_e/ϵ	C_o/ϵ	C_e/ϵ	Δ%	C_o/ϵ	Δ%
1.0	5.06335	7.42715	5.06684	0.069	7.43240	0.070
0.2	1.84964	3.92401	1.85092	0.069	3.92671	0.069
0.1	1.41577	3.16861	1.41675	0.069	3.17079	0.069
0.01	0.85946	1.66447	0.86006	0.069	1.66562	0.069
0.001	0.64785	1.04769	0.64829	0.069	1.04841	0.069
0.0001	0.52309	0.75769	0.52346	0.069	0.75821	0.069

[a]Δ% = $\frac{\text{approximate value - exact value}}{\text{exact value}}$ x 100%.

When the full solution for T>0 was investigated, it was found that when T becomes very small, anomalous behavior is observed. This behavior is not well understood, but it is clear that it is numerical and not physical in nature as the Gaussian quadrature points become very closely packed.

A modified model was devised to represent strips of very small thicknesses. This modification is based on the premise that the charge distributions on the wide sides (upper and lower sides) of a very thin conductor must be very close to the charge distribution that exists on the idealized zero thickness strip with the same geometry. In fact, ideally one would expect the charge distribution on the zero thickness strip to be divided equally between upper and lower surfaces on the nonzero thickness, but very thin, conducting strip. If $\rho_s(t_i)$ equals the wide side surface charge distribution, then,

$$\rho_s(t_i)\Big|_{T>0} = A\ \rho_s(t_i)\Big|_{T=0} \qquad (11)$$

where A might equal 0.5. One can apply this charge distribution to Equation (5) and compute the remaining charge distribution on the so-called thin sides of the conductors. Numerical experience indicates that A=0.4925 is a more optimal value, although this is somewhat subjective.

Figure 2 illustrates the behavior of the numerical solution (Chestnut[1]) of Equation (5) and that of the modified technique for a wide single centered strip. It is an exaggerated example, but illustrative. Cohn's[4] approximate solution is also illustrated. Figure 3 illustrates the behavior of the numerical solution for a specific pair of coplanar strips. It is clear that the modified numerical technique in these cases more nearly reaches the exact solution in the asymptote than does the original Chestnut method (Equation 5). It should be noted that the modified technique appears to break down if T becomes too large. The line of demarcation between the two models is ill defined.

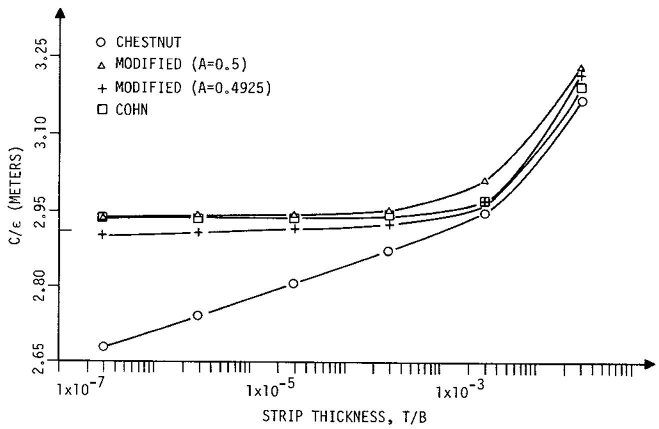

Figure 2. Capacitance of the single centered strip transmission line for a range of thicknesses. Strip width W/B=0.3. Exact zero thickness solution = 2.91347.

The coupling and the characteristic impedance of coupled line can be expressed in terms of the even and odd mode capacitances. Coupling may be defined as

$$C_c = -10\ \log_{10}|k|^2 \text{ in dB} \qquad (12)$$

where k is the voltage coupling coefficient

$$k = \frac{C_o - C_e}{C_o + C_e} \qquad (13)$$

and

$$Z_o = \frac{\epsilon\ \mu_o}{\sqrt{C_o C_e}} \qquad (14)$$

The modified, thin strip, analysis was applied to a 50 Ω directional coupler of a specific geometry. Three standard copper thicknesses were considered. B was chosen as 155.7 mils. S was chosen as 30.7 mils. This geometry is

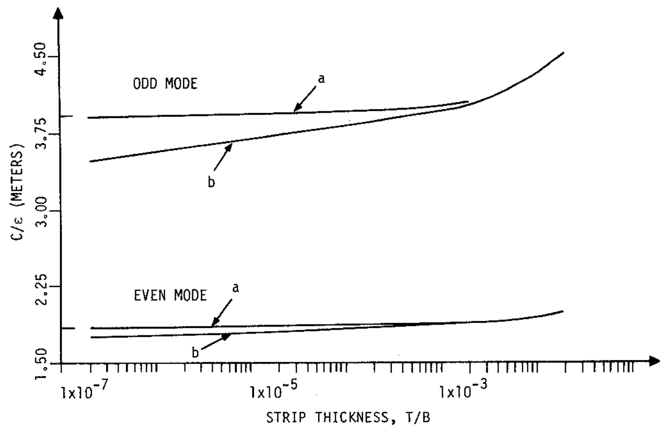

Figure 3. Even and odd mode capacitances of a coplanar pair of coupled strip conductors for a range of thicknesses. Strip widths W/B=0.2 and strip overlap $W_c/B=-0.1$.

Curves a and b are results of the modified (A=0.4925) and Chestnut techniques, respectively. Exact zero thickness solutions indicated by horizontal bars.

based on practical considerations and was suggested by the U. S. Naval Weapons Center, China Lake, California. The standard thicknesses were

2 oz copper, where T=2.8 mils
1 oz copper, where T=1.4 mils
½ oz copper, where T=0.7 mils

Figure 4 illustrates the increase in coupling due to increases in strip thickness. The modified thin strip model was used.

$$\Delta\, dB = |dB_{T>0} - dB_{T=0}| \qquad (15)$$

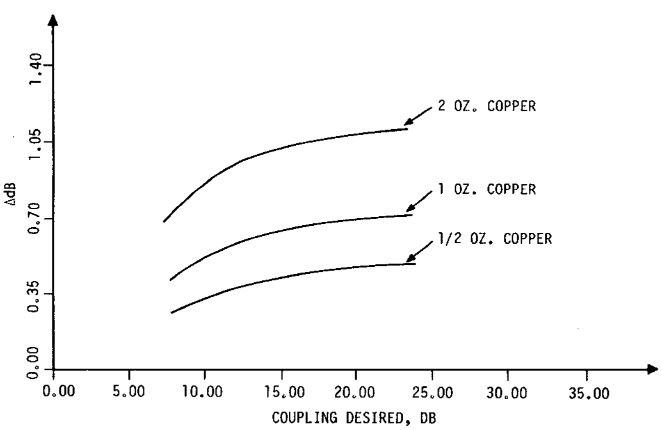

Figure 4. Effect of strip conductor thickness on overlapped coupled striplines S/B = .307/(.125 + .307), $Z_o = 50\ \Omega$, $\epsilon_r = 2.22$

The actual coupling achieved is given by

$$dB_{actual} = dB_{T=0} - \Delta dB \qquad (16)$$

It can be seen that the thinnest of the copper strips produces a significant change in the amount of coupling. The 2 oz copper thickness produces about a 20 percent increase in k. The effect of strip thickness is not negligible.

An interesting curve may be seen in Reference 3, which shows the effects of changing the S/B ratio on ΔdB. The greatest effect on coupling occurs with the largest S/B ratio.

References

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4. Cohn, Seymour B., "Characteristic Impedance of the Shielded-Strip Transmission Line," IEEE Transaction on Microwave Theory and Technique, MTT-2, No. 2 (July 1954): 52-57.